Stretches and Compressions

These notes are intended as a summary of section 3.3 (p. 193 – 200) in your workbook. You should also read the section for more complete explanations and additional examples.

Recall that $f(x)=x^2+2$ is $f(x)=x^2$ translated 2 units up, and that $f(x)=(x+2)^2$ is $f(x)=x^2$ translated 2 units left.

Adding or subtracting a constant results in a translation,

- vertically if the constant is added to the entire function, as in $f(x)+k$
- horizontally if the constant is added to only the x-value, as in $f(x-h)$

In this lesson, we will examine what happens when you multiply by a constant.

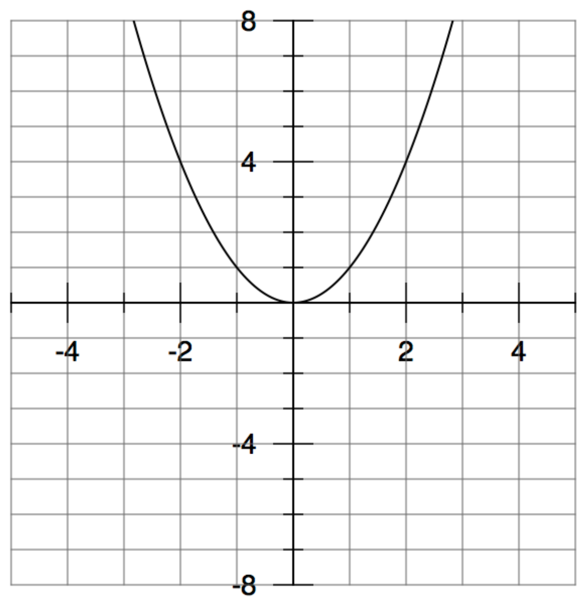
Vertical Stretch/Compression

On the axes pictured to the right, you will see the graph of $y=x^2$.

To see how the graph of $y=ax^2$ compares to the graph of $y=x^2$, plot the following functions on the same axes:

$y=0.5x^2$ $\quad$ $y=-0.5x^2$

$y=2x^2$ $\quad$ $y=-2x^2$



In general, the graph of $y=af(x)$ is the image of the graph of $y=f(x)$ after being vertically stretched, compressed, or reflected.

If $|a|>1$, there is a vertical stretch by a factor of $|a|$.

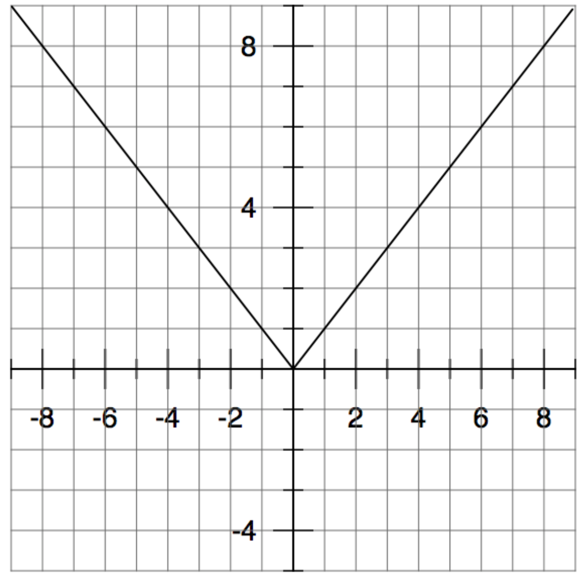
If $0<|a|<1$, there is a vertical compression by a factor of $|a|$.

If $a<0$ there is a reflection in the x-axis, in addition to the stretch or compression.

Note: The easiest way to draw a vertical stretch/compression of a graph is to replace every point (x,y) on $y=f(x)$ by a point (x,ay) on $y=af(x)$.

Example 1 (sidebar p. 196)

Here is the graph of $y = f(x)$. Sketch the graph of $y = -\frac{1}{4}f(x)$. State the domain and range of each function.



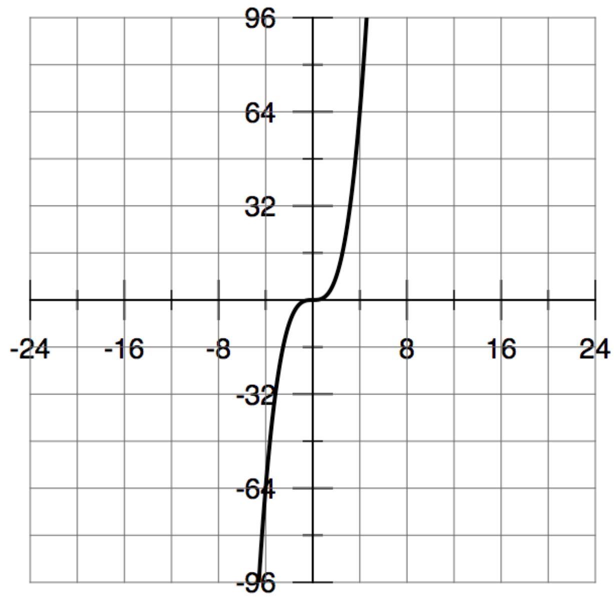
Horizontal Stretch/Compression

On the axes pictured to the right, you will see the graph of $y = x^3$.

To see how the graph of $y = (bx)^3$ compares to the graph of $y = x^3$, plot the following functions on the same axes:

$$y = (2x)^3$$

$$y = \left(\frac{1}{4}x\right)^3 \qquad y = \left(-\frac{1}{4}x\right)^3$$



In general, the graph of $y = f(bx)$ is the image of the graph of $y = f(x)$ after being horizontally stretched, compressed, or reflected.

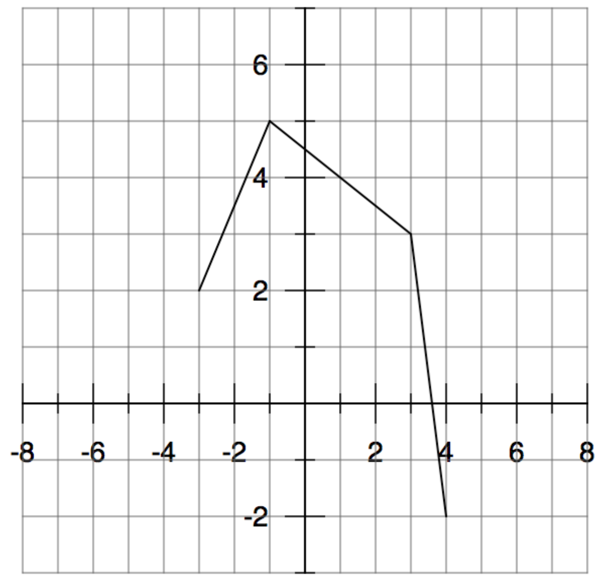
If $|b| > 1$, there is a horizontal compression by a factor of $\frac{1}{|b|}$.

If $0 < |b| < 1$, there is a horizontal stretch by a factor of $\frac{1}{|b|}$.

If $b < 0$ there is a reflection in the y-axis, in addition to the stretch or compression.

Note: The easiest way to draw a horizontal stretch/compression of a graph is to replace every point (x, y) on $y = f(x)$ by a point $\left(\frac{x}{b}, y\right)$ on $y = f(bx)$.

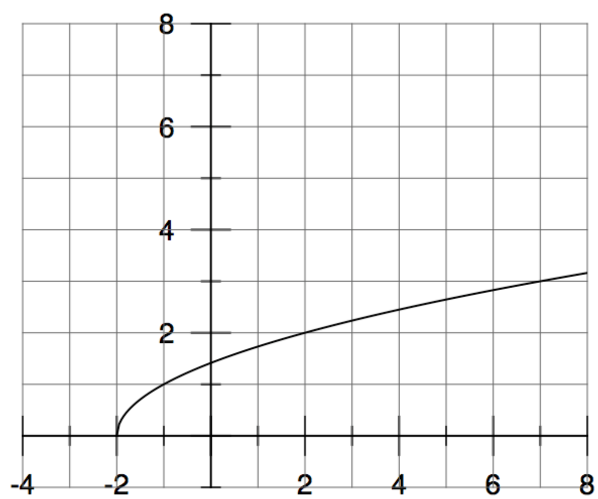
Example 2 (sidebar p. 198)
Here is the graph of $y = g(x)$. Sketch the graph of $y = g(0.5x)$. State the domain and range of each function.



Combining Transformations

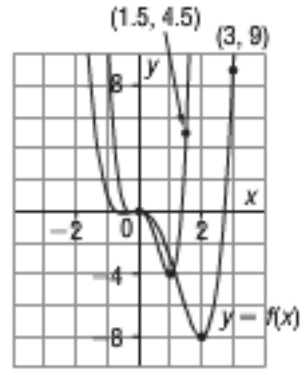
Stretches, compressions, and reflections may be combined. In general, the point (x, y) on $y = f(x)$ becomes the point $\left(\frac{x}{b}, ay\right)$ on $y = af(bx)$.

Example 3 (sidebar p. 199)
Here is the graph of $y = f(x)$. Sketch the graph of $y = 4f(-0.5x)$. State the domain and range of each function.



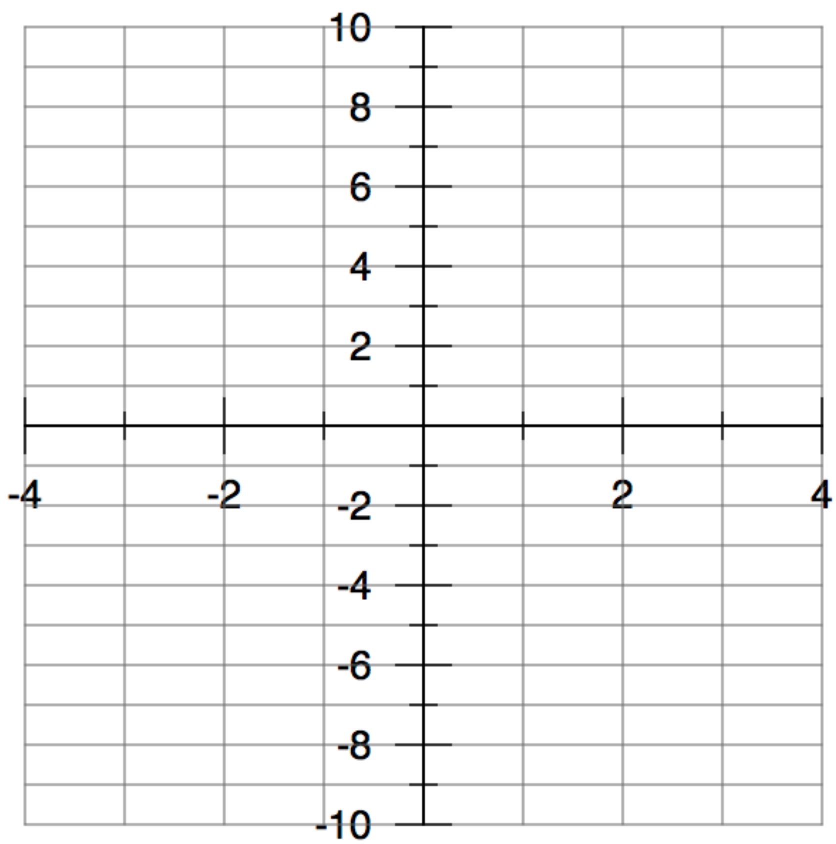
Example 4 (sidebar p. 200)

The graphs of $y = f(x)$ and its image after a vertical and/or horizontal compression are shown. Write an equation of the image graph in terms of the function f.



Example 5 (not in workbook)

Graph $y = 3\left(-\frac{1}{2}x\right)^3$.



Homework: #3 – 5, 7 – 10, 12 – 14 in the exercises (p. 201 – 211). Answers on p. 211.